Aula 15

1) Seja $f(x,y) = \dfrac{\sqrt{x^2+xy-4}}{2y-x}$

a) Determinar o domínio de f e a curva de nível 1 e represente ambos graficamente

C. aux

$x^2 + 4\left(\frac{1}{2}x\right)^2 = 4$

$\Leftrightarrow x^2 + 4 \times \frac{1}{4} x^2 = 4$

$\Leftrightarrow x = \pm\sqrt{2} \rightsquigarrow y = \pm\frac{\sqrt{2}}{2}$

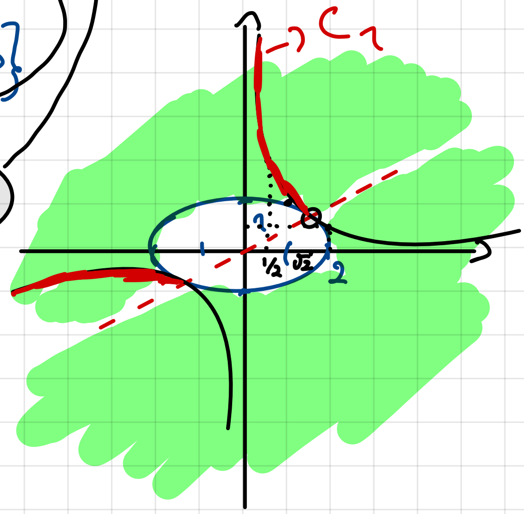
$D_f = \{(x,y) \in \mathbb{R}^2 : x^2+4y^2-4 \geq 0 \wedge 2y-x \neq 0\}$

$\boxed{x^2+4y^2 \geq 4}$ $\quad y \neq \frac{1}{2}x$

$\Leftrightarrow \frac{x^2}{4} + y^2 \geq 1$

$\Leftrightarrow \frac{x^2}{4} + \frac{y^2}{1^2} \geq 1$

x	y
0	0
2	1



$C_1 = \{(x,y) \in D_f : f(x,y) = 1\}$

$\frac{\sqrt{x^2+4y^2-4}}{2y-x} = 1 \Leftrightarrow \sqrt{x^2+4y^2-4} = 2y-x$

$\Leftrightarrow \left(\sqrt{x^2+4y^2-4}\right)^2 = (2y-x)^2 \wedge 2y-x>0$

$\Leftrightarrow \cancel{x^2} + \cancel{4y^2} - 4 = \cancel{4y^2} - 4xy + \cancel{x^2} \wedge 2y > x$

$\Leftrightarrow -4 = -4xy \wedge y > \frac{1}{2}x$

$\Leftrightarrow y = \frac{1}{x}$ $(x \neq 0)$ $\wedge\ y > \frac{1}{2}x$

x	y
1/2	2
1	1
2	1/2

b) Calcular $\frac{\partial f}{\partial x}$ e $\frac{\partial f}{\partial y}$

$\frac{\partial f}{\partial x} = \left(\frac{\sqrt{x^2+4y^2-4}}{2y-x}\right)'_x = \frac{\left(\sqrt{x^2+4y^2-4}\right)'_x \times (2y-x) - \sqrt{x^2+4y^2-4} \times (2y-x)'_x}{(2y-x)^2}$

$\frac{2x}{2\sqrt{x^2+4y^2-4}}$ $\quad = -1$

$= \dfrac{\frac{x(2y-x)}{\sqrt{x^2+4y^2-4}} + \sqrt{x^2+4y^2-4}}{(2y-x)^2}$

$\frac{\partial f}{\partial y} = \dfrac{\frac{4y(2y-x)}{\sqrt{x^2+4y^2-4}} + 2\sqrt{x^2+4y^2-4}}{(2y-x)^2}$

c) Calcule a derivada direcional de f no ponto $(-1,1)$ na direção de $\vec{u}=(2;-3)$

f é diferenciável em $(-1,1)$ pois as derivadas parciais existem numa bola centrada nesse ponto e são contínuas nesse ponto.

$\|\vec{u}\| = \sqrt{2^2+(-3)^2} = \sqrt{13} \neq 1 \longrightarrow \frac{\vec{u}}{\|\vec{u}\|} = \left(\frac{2}{\sqrt{13}} ; \frac{2}{\sqrt{13}}\right)$

C. aux

$\sqrt{x^2+4y^2-4} \rightarrow \sqrt{(-1)^2+4\times 1^2-4} = 1$

$2y-x \rightarrow 2\times 1-(-1) = 3$

$\rightarrow \frac{\partial f}{\partial x}(-1,1) = \frac{\frac{-1\times 3}{1}+1}{3^2} = -\frac{2}{9}$

$\frac{\partial f}{\partial y}(-1,1) = \frac{\frac{4\times 1\times 3}{1} - 2\times 1}{3^2} = \frac{10}{9}$

$\nabla f(-1,1) = \left(\frac{-2}{9} ; \frac{10}{9}\right)$

$D_{\vec{u}} f(-1,1) = \nabla f(-1,1) \cdot \frac{\vec{u}}{\|\vec{u}\|}$

$= \left(-\frac{2}{9} ; \frac{10}{9}\right) \cdot \left(\frac{2}{\sqrt{13}} ; \frac{-3}{\sqrt{13}}\right)$

$= -\frac{2}{9} \times \frac{2}{\sqrt{13}} + \frac{10}{9} \times \frac{(-3)}{9} \times \frac{(-3)}{\sqrt{13}} = \frac{-34}{9\sqrt{13}}$

d) Escreve a equação do plano tangente e da reta normal ao gráfico de f no ponto $(-1; 1; 1/3)$ $\rightarrow f(-1;1)$

Plano tangente:

$$Z = f(-1;1) + \frac{\partial f}{\partial x}(-1;1) \times (x-(-1)) + \frac{\partial f}{\partial y}(-1;1) \times (y-1)$$

$$\Leftrightarrow Z = \frac{1}{3} + \left(\frac{-2}{9}\right) \times (x+1) + \frac{10}{9} \times (y-1)$$

$$\Leftrightarrow Z = \frac{1}{3} - \frac{2}{9}x - \frac{2}{9} + \frac{10}{9}y - \frac{10}{9}$$

$$\Leftrightarrow Z = -\frac{2}{9}x + \frac{10}{9}y - 1$$

Reta normal:

$$(x,y,z) = (-1;1;1/3) + \lambda\left(\frac{-2}{9}; \frac{10}{9}; -1\right), \lambda \in \mathbb{R}$$

2) Calcule, caso existam, os seguintes limites:

a) $\lim\limits_{(x,y)\to(-2,0)} \frac{(x+2)^3 xy}{(x+2)^2+y^2}$ $\left(\frac{0}{0}\right)$

$= \lim\limits_{(x,y)\to(-2,0)} (x+2)xy \times \frac{(x+2)^2}{(x+2)^2+y^2} = 0$

$(x+2)xy \to 0$; $\frac{(x+2)^2}{(x+2)^2+y^2}$ limitada; Slide 21

$0 \leq (x+2)^2 \leq (x+2)^2+y^2$

$\div (x+2)^2+y^2$

$0 \leq \frac{(x+2)^2}{(x+2)^2+y^2} \leq 1$

$\hookrightarrow$ função limitada

b) $\lim\limits_{(x,y)\to(-2,0)} \frac{(x+2)\times y}{(x+2)^2+y^2}$ $\left(\frac{0}{0}\right)$

Seja $A_m = \{(x,y) \in \mathbb{R}^2 : x+2 = my, y \neq 0\}$

$(-2,0)$ é ponto de acumulação de A_m

$\lim\limits_{\substack{(x,y)\to(-2,0) \\ x+2=my}} \frac{(x+2)\times y}{(x+2)^2+y^2} = \lim\limits_{y\to 0} \frac{my \times y}{(my)^2+y^2} = \lim\limits_{y\to 0} \frac{my^2}{m^2y^2+y^2}$

$= \lim\limits_{y\to 0} \frac{m\cancel{y^2}}{\cancel{y^2}(m^2+1)} = \lim\limits_{y\to 0} \frac{m}{m^2+1}$

Como este limite depende de m então o limite inicial não existe